**ANNAPOLIS VALLEY REGIONAL SCHOOL BOARD**

# Pre-Calculus 12

## Sample Examination Questions

January 2016

# ***Selected Response Questions***

1. The inverse function of $f(x) = y = x^2 + 1,\ x \leq 0$ is:

| | | | |
|---|---|---|---|
| A | $f^{-1}(x) = \pm\sqrt{x-1}, x \geq 1$ | B | $f^{-1}(x) = \sqrt{x-1}, x \geq 1$ |
| C | $f^{-1}(x) = -\sqrt{x-1}, x \geq 1$ | D | $f^{-1}(x) = -\sqrt{x-1}, x \geq 0$ |

2. The graph of $y = f(x)$ is stretched horizontally by a factor of $\frac{1}{3}$. Determine the equation of the transformed graph.

| | | | |
|---|---|---|---|
| A | $y = 3f(x)$ | B | $y = \frac{1}{3}f(x)$ |
| C | $y = f(3x)$ | D | $y = f(\frac{1}{3}x)$ |

3. The point $P(-2,-6)$ is on the graph of $y = f(x)$. Which point must be on the graph of $y = -f(x-3)$?

| | | | |
|---|---|---|---|
| A | $(-1,-6)$ | B | $(-5,-6)$ |
| C | $(-1,6)$ | D | $(1,6)$ |

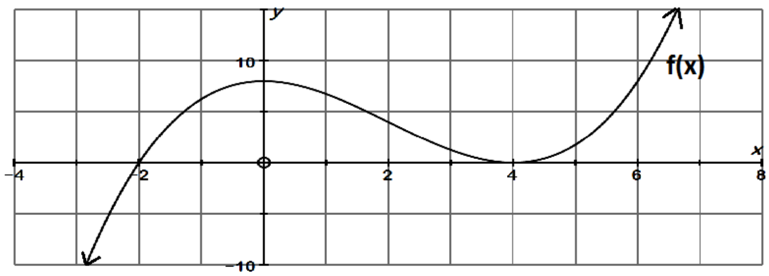
4. What is the mapping rule for: $y = -\frac{1}{3}f(\frac{1}{3}x + 3)$?

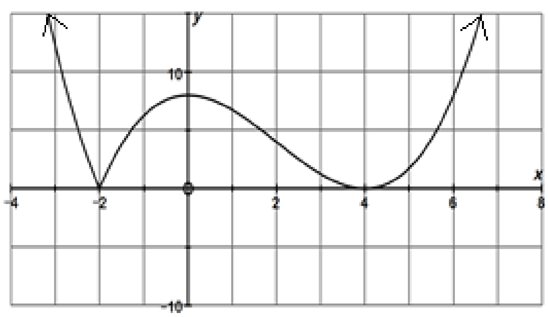
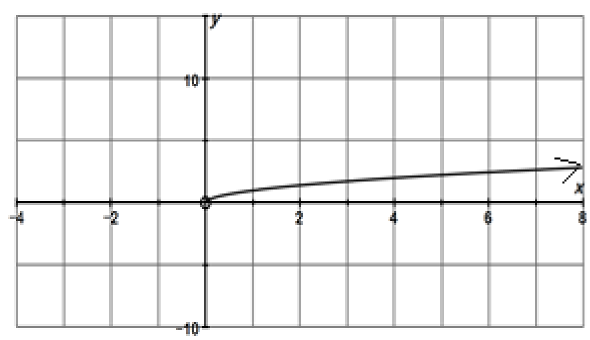
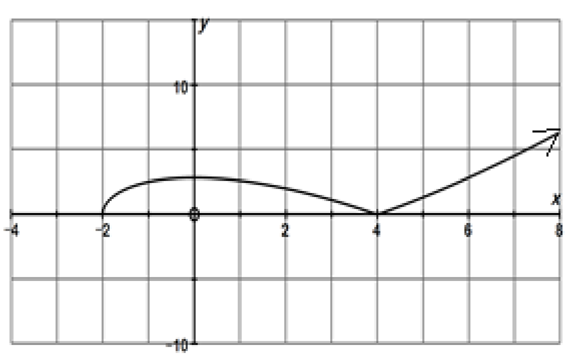
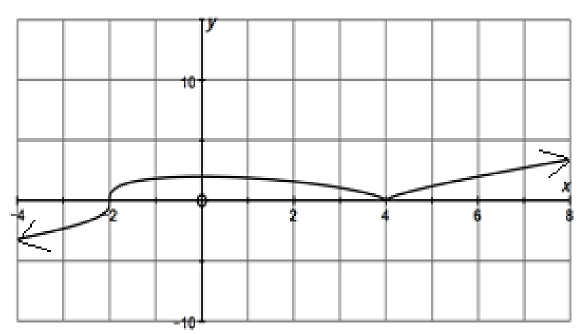
| | | | |
|---|---|---|---|
| A | $(x,y) \to (\frac{1}{3}x - 3, -\frac{1}{3}y)$ | B | $(x,y) \to (\frac{1}{3}x - 9, -\frac{1}{3}y)$ |
| C | $(x,y) \to (3x - 3, -\frac{1}{3}y)$ | D | $(x,y) \to (3x - 9, -\frac{1}{3}y)$ |

5. Consider the graphs of the functions $f(x) = x^2 - 4$ and $g(x) = \sqrt{f(x)}$. What is the domain and range of $g(x)$?

| | | | |
|---|---|---|---|
| A | $x \in (-\infty, -2] \cup [2, \infty)$<br>$y \in [-4, \infty)$ | B | $x \in R$<br>$y \in [0, \infty)$ |
| C | $x \in [-2, 2]$<br>$y \in [-4, \infty)$ | D | $x \in (-\infty, -2] \cup [2, \infty)$<br>$y \in [0, \infty)$ |

6. Given the graph of $y = f(x)$ as shown, determine the graph of $y = \sqrt{f(x)}$.




| | | | |
|---|---|---|---|
| A |  | B |  |
| C |  | D |  |

7. Given the original function $y = \sqrt{x}$ and stretching it vertically by a factor of 3 and horizontally by a factor of 2, then translating it 1 unit to the right, what would be the resulting function?

| | | | |
|---|---|---|---|
| A | $y = \frac{1}{3}\sqrt{\frac{1}{2}(x-1)}$ | B | $y = 3\sqrt{\frac{1}{2}x - \frac{1}{2}}$ |
| C | $y = 3\sqrt{\frac{1}{2}x - 1}$ | D | $y = \frac{1}{3}\sqrt{\frac{1}{2}(x+1)}$ |

8. Determine the exact value(s) of k in the following polynomial $y = kx^3 - k^2x + 14$ if the remainder is +38 when divided by x+2.

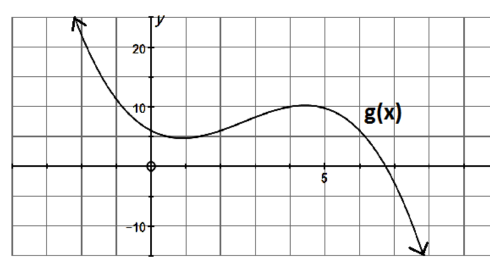
| | | | |
|---|---|---|---|
| A | $k = 2\ and\ k = 6$ | B | $k = -2\ and\ k = 6$ |
| C | $k = 2\ and\ k = -6$ | D | $k = -2\ and\ k = -6$ |

9. Which of the following functions have a root with multiplicity of 2?

I. $y = x^3 + 5x^2 + 8x + 4$
II. $y = 2x^3 + 11x^2 + 18x + 9$
III. $y = x^5 + 11x^4 + 46x^3 + 90x^2 + 81x + 27$

| | | | |
|---|---|---|---|
| A | $I, II\ and\ III$ | B | $I\ and\ II$ |
| C | $I\ and\ III$ | D | $none$ |

10. A function in the form $g(x) = ax^3 + bx^2 + cx + d$ is graphed to the right. What are the characteristics of *a* and *d* in the function?



| | | | |
|---|---|---|---|
| A | $a > 0\ and\ d > 0$ | B | $a < 0\ and\ d > 0$ |
| C | $a > 0\ and\ d < 0$ | D | $a < 0\ and\ d < 0$ |

11. The angle measuring $\frac{-2\pi}{5}$ radians is coterminal to what angle in standard position?

| | | | |
|---|---|---|---|
| A | 288° | B | 72° |
| C | 108° | D | 144° |

12. In high school, a shot put is thrown out of a circle with a diameter of 9 feet. A curved wooden stopboard is placed in an arc around part of this circle. The central angle is 72°. Determine the length of the curved stopboard.

| | | | |
|---|---|---|---|
| A | 2.83 ft. | B | 5.66 ft. |
| C | 11.31 ft. | D | 16 ft. |

13. Determine an expression for all angles co-terminal with a standard position angle measuring 510°. Express your answer in radians.

| | | | |
|---|---|---|---|
| A | $\frac{11\pi}{6} + \pi k, k \in Z$ | B | $\frac{11\pi}{6} + 2\pi k, k \in Z$ |
| C | $\frac{5\pi}{6} + \pi k, k \in Z$ | D | $\frac{5\pi}{6} + 2\pi k, k \in Z$ |

14. A point with an x value of 3 lies on the circle with equation $x^2 + y^2 = 10$. This point also lies on the terminal arm of $\theta$ in standard position. Determine the value of $cos\theta$.

| | | | |
|---|---|---|---|
| A | $\frac{\sqrt{10}}{3}$ | B | $\frac{10}{3}$ |

| | | | |
|---|---|---|---|
| C | $\frac{3\sqrt{10}}{10}$ | D | $\frac{3}{10}$ |

15. Given $cos(\Theta) = \frac{4}{5}$ in the fourth quadrant, what is the approximate value of $y = csc(\Theta)$.

| | | | |
|---|---|---|---|
| A | $-0.6$ | B | $-\frac{5}{3}$ |
| C | $\frac{5}{4}$ | D | $-0.75$ |

16. What is the equation of a circle, centre at the origin, that has (-6, 8) as a point on the circle.

| | | | |
|---|---|---|---|
| A | $x^2 + y^2 = \sqrt{(28)}$ | B | $x^2 + y^2 = 10$ |
| C | $x^2 + y^2 = 28$ | D | $x^2 + y^2 = 100$ |

17. Determine the y-intercept of $y = 2^x$ with the following transformations: a reflection about the y-axis, vertical translation up 2 and horizontal translation left 1.

| | | | |
|---|---|---|---|
| A | $(0, -4)$ | B | $(0,2.5)$ |
| C | $(0,1)$ | D | $(-1, 3)$ |

18. Given $y = 3log(x + 5) - 2$ determine the equation of the asymptote.

| | | | |
|---|---|---|---|
| A | $\mathrm{x} = -5$ | B | $\mathrm{x} = 5$ |
| C | $\mathrm{y} = -2$ | D | $\mathrm{y} = 2$ |

19. Which expression is a simplified form of $2log_2(x) + 0.5log_2(y) + 3$ ?

| | | | |
|---|---|---|---|
| A | $log_2(xy) + 3$ | B | $log_2\left(2x + \frac{y}{2}\right) + 3$ |
| C | $log_2\left(8x^2\sqrt{y}\right)$ | D | $\left(log_2(x^2)\right)\left(log_2\left(y^{\frac{1}{2}}\right)\right) + 3$ |

20. Deborah bought a minivan which is known to depreciate by 20% every 2 years. If the purchase price was $27 000, which equation will model the value (V) in dollars of the vehicle after n years.

| | | | |
|---|---|---|---|
| A | $V = 27\,000(0.2)^{n/2}$ | B | $V = 27\,000(0.2)^{2n}$ |
| C | $V = 27\,000(0.8)^{n/2}$ | D | $V = 27\,000(0.8)^{2n}$ |

21. Determine the end behaviour of: $y = \frac{4x-3}{x+1}$

| | | | |
|---|---|---|---|
| A | $y \to 4$ | B | $y \to -3$ |
| C | $y \to -1$ | D | $y \to 0$ |

22. Determine the restricted values of: $f(x) = \frac{x+2}{x^2+6x+8}$ and classify the nature of those restrictions.

| | | | |
|---|---|---|---|
| A | P.O.D @ $(-4,0)$ and<br>V.A: $x = -2$ | B | P.O.D @ $(-2, 0.5)$ and<br>V.A: $x = -4$ |
| C | V.A: $x = -4$ and<br>V.A: $x = -2$ | D | V.A: $x = 2$ and<br>V.A: $x = 4$ |

23. Choose the correct domain and range for $f(x) = 8 + \frac{3}{x-2}$

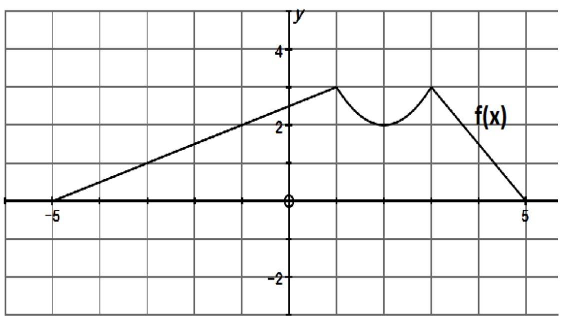
| | | | |
|---|---|---|---|
| A | $D: x \neq 2, x \in \mathbb{R}$;<br>$R: y \neq 0, y \in \mathbb{R}$ | B | $D: x \neq 2, x \in \mathbb{R}$;<br>$R: y \neq 8, y \in \mathbb{R}$ |

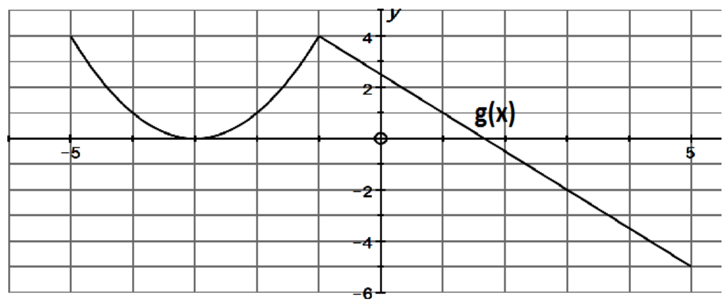
| | | | |
|---|---|---|---|
| C | $D: x \neq -2, x \in \mathbb{R}$<br>$R: y \neq 0, y \in \mathbb{R}$ | D | $D: x \neq 8, x \in \mathbb{R}$<br>$R: y \neq 2, y \in \mathbb{R}$ |

24. Given $g(x) = x^2$ and $h(x) = \sin(x)$, determine $h(g(x+1))$.

| | | | |
|---|---|---|---|
| A | $\sin(x+1)^2$ | B | $[\sin(x+1)]^2$ |
| C | $(sinx)^2 + 1$ | D | $\sin(x)^2 + 1$ |

25. The graphs of $y = f(x)$and $y = g(x)$ are graphed below.
    What is the value of $f(g(-1))$.




| | | | | |
|---|---|---|---|---|
| A | $1.5$ | | B | $-0.5$ |
| C | $2$ | | D | $-3.5$ |

26. Consider the functions $f(x) = x^2 + 7x + 6$ and $g(x) = x^2 - 1$. Identify any non-permissible values for the function, $h(x) = \frac{g(x)}{f(x)}$.

| | | | |
|---|---|---|---|
| A | $x \neq 1, 6$ | B | $x \neq -1, -6$ |
| C | $x \neq -1$ | D | $x \neq -6$ |

27. A man chooses 5 or 6 movies from his friend's list of 40 movies of all time to watch. Which expression gives the total number of possible orders in which the man could view the movies that he selects?

| | | | |
|---|---|---|---|
| A | ${}_{40}C_5 + {}_{40}C_6$ | B | ${}_{40}C_5$ X ${}_{40}C_5$ |
| C | ${}_{40}P_5 + {}_{40}P_6$ | D | ${}_{40}P_5$ X ${}_{40}P_6$ |

28. A town council is made up of 16 members including the mayor. They decide to form a sub-committee of 5 to consider environmental issues. If the mayor must be one of the members of this committee, how many different committees are possible?

| | | | |
|---|---|---|---|
| A | ${}_{16}C_5$ | B | ${}_{16}P_5$ |
| C | ${}_{15}C_4$ | D | ${}_{15}P_4$ |

29. Which of the following is the simplification of $\frac{(n+2)!}{n!}$ ?

| | | | |
|---|---|---|---|
| A | $n(n+2)$ | B | $(n+2)(n+1)$ |
| C | $2!$ | D | $n$ |

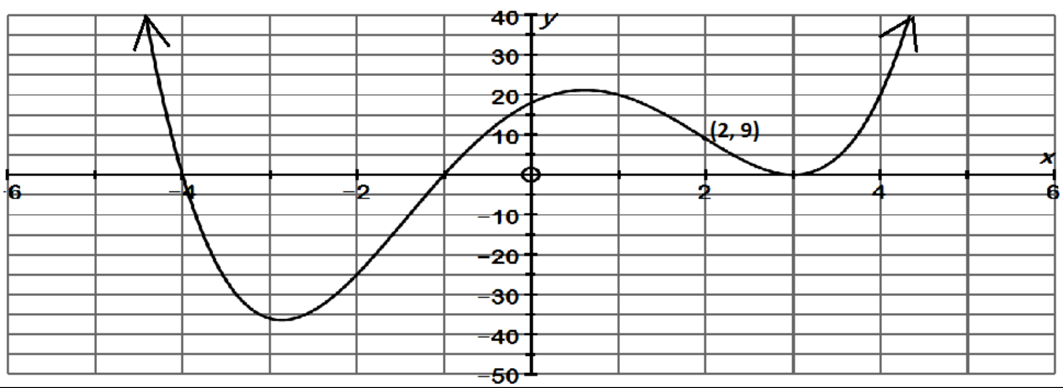
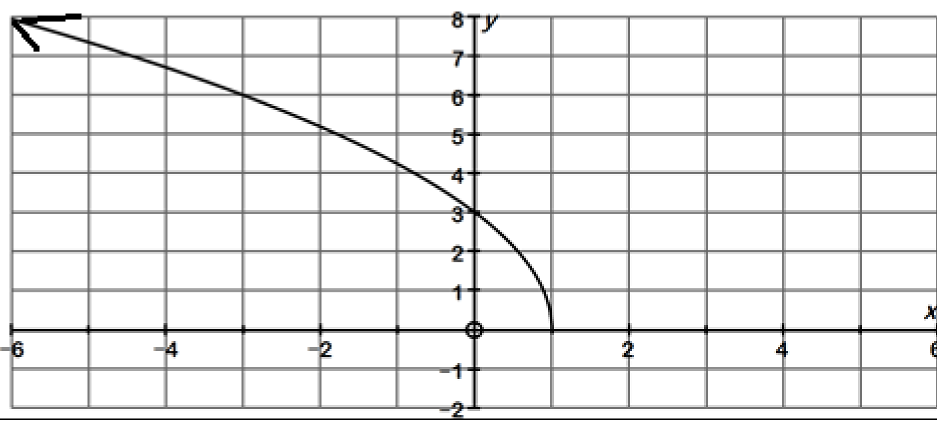
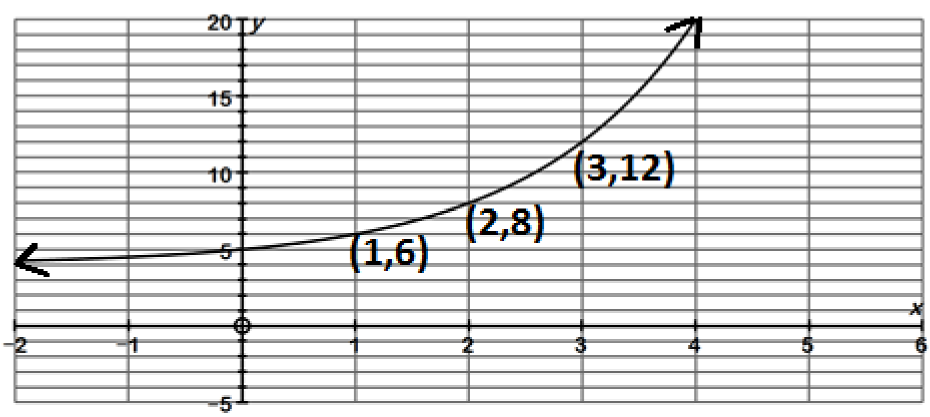
30. What is the fourth term of the expression $(4x-y)^6$ ?

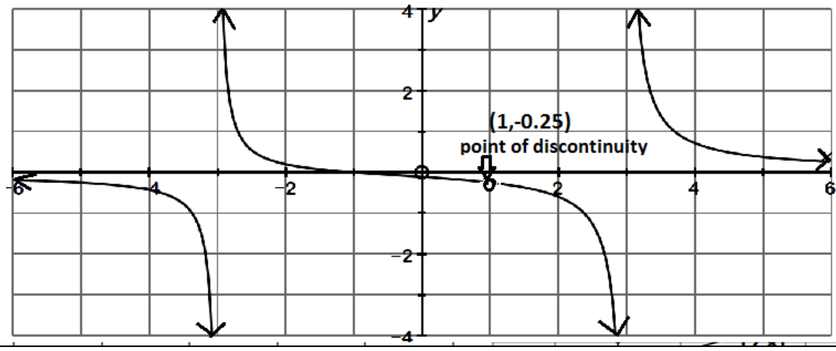
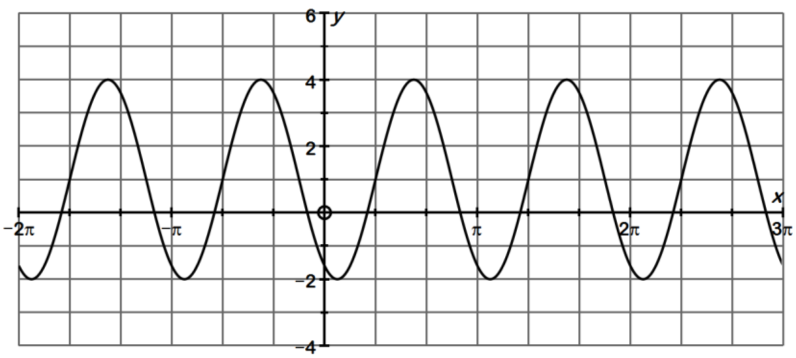
| | | | |
|---|---|---|---|
| A | ${}_6C_4(4x)^3(-y)^3$ | B | ${}_6C_3(4x)^3(-y)^3$ |
| C | ${}_6C_4(4x)^4(-y)^2$ | D | ${}_6C_3(4x)^4(-y)^2$ |

## *Sample* Full Solution Questions

## Question 31 (Determine Equations from graphs)

Determine the functions graphed below:

| | |
|---|---|
| a) |  |
| b) |  |
| c) |  |

| | |
|---|---|
| d) |   |
| e) |   |

## Question 32 (Algebraically solve equations for all solutions)

Solve each of the following equation for all solutions.

a) $\sin^2 2x + \cos^2 x = 0$

b) $2\sin^2 x + \cos x - 1 = 0$

c) $2\cos(2x)\cos\left(\pi/5\right) + 2\sin(2x)\sin\left(\pi/5\right) = 1$

d) $\dfrac{2!n!}{4!(n-5)!} = \dfrac{(n-1)!}{(n-4)!}$

e) $\dfrac{x!}{(x-2)!} = 20$

f) $\log_2(5x-2) - \log_2 2 = \frac{1}{2}\log_2 36 + 2\log_2 3$

g) $\ln(x+1) + \ln(x-2) = \ln(4)$

h) $\frac{1}{3}\log_3 27 + \log_3 x = 4^{\frac{1}{2}}$

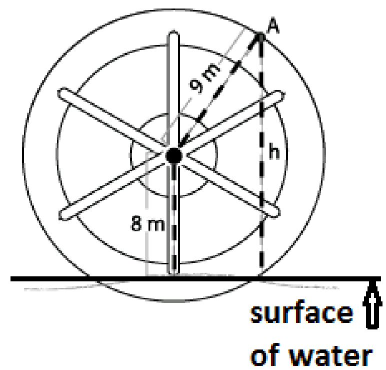
i) $3\log_4(2x+1) - \log_4(x-2) = 1$

j) $3^{x+2} - 4 = 6$

k) $4(2^x)^3 + 2(2^x) - 18 = 40(2^x)^2 + 2$

## Question 33 (Sinusoidal Applications)

1. A waterwheel rotates at 6 revolutions per minute (rpm). After 2 seconds, point A is at its greatest height above the water.




   a) Sketch a graph of the distance of point A from the surface of the water in terms of time(t).

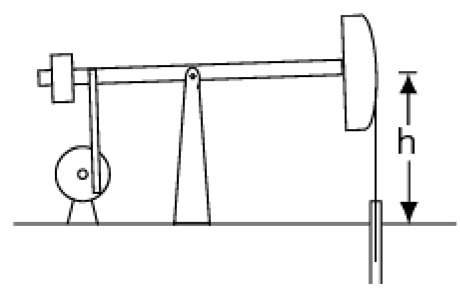
   b) Find the equation of the sinusoidal axis, the amplitude and the period of the function.

   c) Write a trigonometric equation to describe the function

   d) Where will the point A on the waterwheel be at 135 seconds?

   e) Determine the first three times when point A reached a height 10 feet above the surface of the water.

2. The jack on an oil well goes up and down, pumping out of the ground. As it does so, the height, *h*, in meters, varies sinusoidally with time, *t*, in seconds. At *t =1* seconds, *h* is at its maximum, 3.7 meters. At *t = 4* seconds, *h*, is at its minimum, 1.5 m.




   a) Draw a graph of *h* versus *t*. Be sure to label all values on both axes.

   b) Write a sinusoidal equation to represent this situation.

   c) Find *h* (to the nearest tenth) when:
      i) $t = 5.5$ seconds
      ii) $t = 9.3$ seconds

   d) When will the height reach 2 metres?

3. A car alarm system is designed so that the alarm sound's frequency is given by the equation $\frac{1}{6}(y-1) = \sin 90x$, where *x* represents time(s) and *y* represents sound frequency (khz). If the car's window is broken, the sound will become twice as fast, with the maximum pitch increasing by 4 khz and the minimum pitch by 2 khz. Write the equation describing the alarm sound after the window is broken.

## Question 34 (Exponential /Logarithmic Applications)

1. Earth's population is increasing by approximately 2.3% each year. The population today is approximately 7.39 billion.
   a) State the function that can be used to describe the population, P (in billions), of Earth after t years.
   b) Algebraically determine how long it will take Earth's population to reach 10.1 billion people. Illustrate your answer graphically.

2. For a 10 year period, [0,10] , the population of a district, in thousands, can be reasonably modelled by the model $P = 23 + \ln(4t - 12)$.
   a) Determine the population 2.5 years into this 10 year period.
   b) Determine when the population will reach 26400.

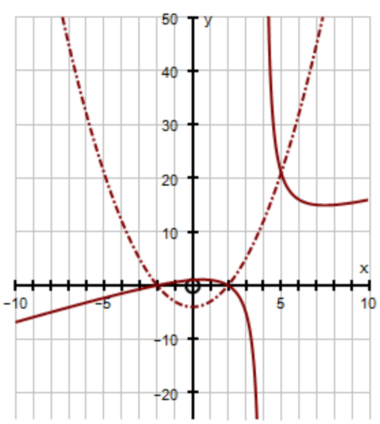
3. Compound Interest: $A = A_o(1 + i)^n$

   i = interest rate / compounding period
   $A_o$ = principle amount
   n = # of compounding periods
   A = total value

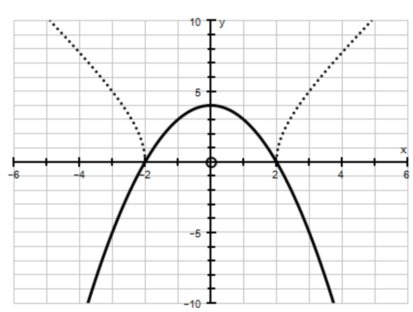
   a) A student deposits $3500 into a bank that pays 6% per annum compounded monthly. How much money will the student have after 8 years?
   b) Algebraically determine how long will it take (in years) for an investment to double if it is invested at 6% compounded monthly?

## Question 35 (Graphically solve equations and justify algebrically)

1. The graphs of $f(x) = x^2 - 4$ and $g(x) = \frac{x^2-4}{x-4}$ are shown to the right. Algebraically determine the intersection points of the curves.




2. Solve algebraically for the intersection point(s) of $f(x) = \sqrt{5x^2 - 20}$ and $g(x) = 4 - x^2$.



## Question 36 (Prove Identities)

Prove the following identities.

a) $(\cot^2 x)(\cos^2 x) = \cot^2 x - \cos^2 x$

b) $\sin 3x = 3\sin x - 4\sin^3 x$

c) $\dfrac{\sin^3 x}{\cos x} + \sin x \cos x = \tan x$

d) $\dfrac{1}{\cos\theta} - \cos\theta = \sin^2\theta \sec\theta$

e) $\cos^4 x - \sin^4 x = \cos 2x$

f) $\dfrac{\sin 2x}{1+\cos 2x} = \tan x$

g) $\csc x - \cos x \csc x = \dfrac{\sin x}{1+\cos x}$

h) $\dfrac{\cos\theta}{1-\sin\theta} = \sec\theta + \tan\theta$

## Question 37 (Solving equation algebraically- for a restricted domain)

1. Algebraically find the roots of $f(x) = 2x^3 + 5x^2 - 5x + 1$. Present your solution in a well-documented manner.

2. Without the use of technology, determine any points of intersection between $f(x) = x^3 - 3x^2$ and $g(x) = 4x - 12$.

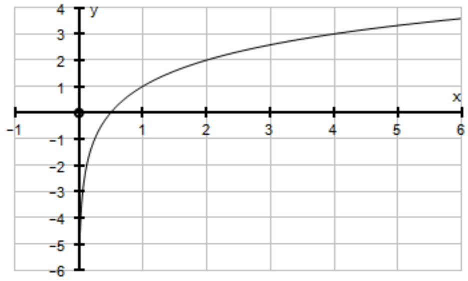
3. Algebraically solve $3\sin(2x) + 5 = 7$ for all values between $-180°$ and $360°$..

4. Without using technology, solve $3sec^2x = 12$ for values between 0 and $2\pi$.

5. Without using technology, solve $sin^2x = 3sinx - 2$ for values between 0 and $3\pi$ .

## Question 38 (Analysis)

1. The graph of $f(x) = (log_2 x) + 1$ is shown below.



Prove algebraically that the graphs of $g(x) = log_2(4x) - 1$ and $h(x) = log_2(2x)$ are equivalent.

2. If *A* is an obtuse angle and $\sin A = \frac{3}{5}$, and if *B* is an acute angle and $\sin B = \frac{12}{13}$, find the exact value for $\tan(A-B)$.

3. Using the laws of logarithms and the definition of an arithmetic sequence, show that is an $\log(a), \log(ab), \log(ab^2), \log(ab^3)$arithmetic sequence. [Recall that an arithmetic sequence is one where terms have a common difference such as 3,5, 7, 9, …]

4. Solve the equation: $\sqrt{2^x} - \frac{12}{\sqrt{2^x}} = 1$.

5. Solve the system $\begin{Bmatrix} y = \log_2(2x-6) \\ y = \log_4 x \end{Bmatrix}$.

6. If $g(x) = 1 - 3x$ and $f\left(g(x)\right) = 9x^2 - 6x + 5$, find the value of $f(1)$.

7. The function $P(x) = 3x^3 - 6ax^2 - 4ax + 8a$ has a root of $2a$, a > 0 *find its other roots.*

8. Prove that: $\frac{e^{-x}}{e^{-x}+1} + \frac{e^x}{e^x+1} = 1$

9. In $\Delta ABC$, $\sin B = \frac{3}{5}$, and $\sin C = \frac{1}{4}$ determine the ratio AB:AC

10. When a colony of wasps was studied, its population was found to be approximated by the model $P(t) = 50e^{0.1t}, t \geq 0$, where P is the population of wasps and t days is the time from the start of the study. Over the same period of time a second wasp colony was also studied. Its population, Q, was found to be approximated by the model $Q(t) = 500 - 450e^{-0.1t}, t \geq 0$. Determine the exact time when these two wasp colonies would have the same population.